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ORIGINAL PAPER

# Cumulative culture and explosive demographic transitions

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**Abstract** A demographic transition is a change in the pattern of growth of a population. Human history records several kinds of such transitions, e.g., from stability to growth or between different kinds of growth. Culture is often implied as the main fuel of demographic transitions, but theorizing is so far limited to verbal arguments. Here we study two simple formal models in which population size and the amount of culture in a population influence each other's dynamics. The first model has two regimes: an equilibrium regime in which both population size and amount of culture reach stable values, and an explosive regime in which both variables increase exponentially without bound. A transition between these regimes is caused by changes in parameters that describe the accuracy of cultural transmission and the interaction between demography and culture. The second model suggests that a transition from extensive to intensive accumulation of culture may derive from a qualitative change in how individuals cooperate to create culture.

**Keywords** Demographic transition · Cumulative culture · Extensive and intensive growth · Mathematical modeling

## 1 Introduction

Demographic transitions, defined as changes in the pattern of growth of a population, have been investigated in as diverse subjects as anthropology, history, demography,

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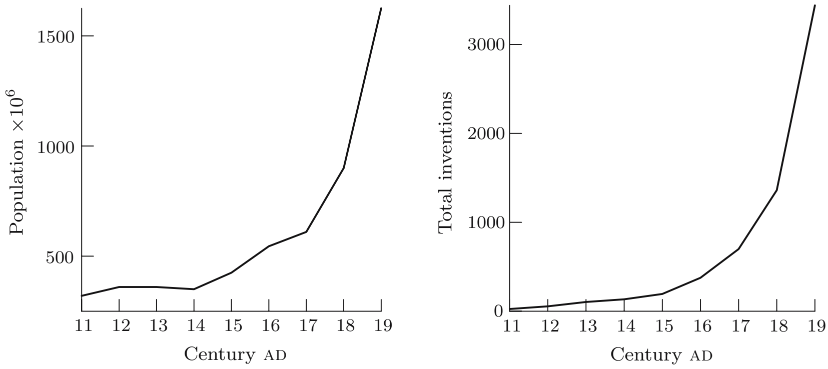


**Fig. 1** Exponential increase in population size (*left*) and number of scientific and technological innovations (right) between 1100 and 1900AD. LEFT: DATA FROM McEvedy and Jones (1978); RIGHT: DATA FROM Darmstaeder and Dubois Reymond (1904)

economics, and evolutionary psychology (Goudsblom et al. 1996; Borgerhoff Mulder 1998). We consider here transitions from a state of demographic equilibrium (constant population size) to one of demographic expansion (population growth). Demographic expansion seems a constant of our species as far back as we can look (Jones 1996b; McEvedy and Jones 1978). This stands in sharp contrast with non-human primates, whose numbers, as far as we know, do not show such a tendency to increase. Thus, a demographic transition from equilibrium to expansion seems to accompany hominization. Another thing, of course, happened at the dawn of our species: human culture. Many animals are capable of maintaining traditions, but human culture is strikingly different. It is, for instance *cumulative*, i.e., it typically increases over generations to levels that no single generation could achieve. Recorded animal traditions, in contrast, are so simple that they could have been established within a single generation, and often by a single creative individual. The ability to accumulate cultural information, we argue in this paper, may have fueled the demographic expansion of hominids.

That demography and culture may influence each other has been suggested many times in anthropology and sociology, at least since the work of Herbert Spencer (reviewed in Carneiro 2003). Empirically, periods of population expansion seem to correspond to periods of significant cultural growth. For instance, the expansion out of Africa of *Homo sapiens* 40,000 years ago is characterized by both demographic growth and the invention of new kinds of stone tools (Klein 1989; Rogers and Harpending 1992; Rogers 1995). In modern times there seems to be a steady trend of demographic and cultural growth (Fig. 1, see also Lehman 1947; Ogburn 1950; McEvedy and Jones 1978).

Here we study formally the interaction between demography and culture, coupling a model of cumulative culture (Enquist and Ghirlanda 2006, Submitted) with a model of population growth borrowed from ecology (Gotelli 2001). We show that two regimes are possible: an equilibrium regime in which both population size and amount of culture reach stable values, and an explosive regime in which both variables increase exponentially without bound. A transition between these regimes may be caused by changes in the accuracy of cultural transmission and in how strongly demography and culture interact. We then extend the model to distinguish between *extensive* growth of culture (proportional to population size) and *intensive* growth (growth in amount of culture per individual, see below).

## 2 Model

We treat population size and culture as macroscopic variables that influence each other's dynamics. Formally, we consider two coupled first-order differential equations in the variables $n$, population size, and $x$, a suitably defined *amount of culture*. Both variables are constrained to be $\geq 0$. Following previous work, we assume that the amount of culture changes as the result of two processes: one that destroys culture and one that creates culture (Enquist and Ghirlanda 2006, Submitted). The first process may represent errors in the transmission of culture; we assume that culture is lost at a rate $\lambda > 0$. The second process represents the creation of culture by individuals. We assume here that each individual creates an amount of culture $\delta > 0$ per unit time, so that culture is created at a rate $\delta n$. We get thus the following dynamics for $x$:

$$\dot{x} = -\lambda x + \delta n, \tag{1}$$

where $\dot{x}$ is the time derivative (rate of change) of $x$, $\dot{x} = \mathrm{d}x/\mathrm{d}t$.

We now derive a dynamics for population size, $n$, starting from ecological models of population growth in the presence of limited resources. In a classical such model, an environment is characterized by a *carrying capacity*, $n_{\max}$, i.e., the maximum number of individuals that can survive in that environment (Gotelli 2001). Demography is then described by the so-called *logistic equation*

$$\dot{n} = rn\left(1 - \frac{n}{n_{\max}}\right), \tag{2}$$

where $r$ is a growth rate parameter. For constant $n_{\max}$ and for any initial size $n_0 > 0$, Eq. 2 yields an asymptotic approach to $n_{\max}$. We establish a connection between demography and culture assuming that culture is able to raise the carrying capacity of the environment, e.g., allowing to gain more resources from the same environment (Rogers 1992). A cultural innovation such as a knife, for instance, may improve the efficiency of food gathering and processing. The simplest possibility is that $n_{\max}$ be a linear function of $x$:

$$n_{\max} = \alpha x + \beta, \tag{3}$$

where $\beta > 0$ is the carrying capacity in the absence of culture and $\alpha > 0$ measures the impact of culture on the carrying capacity: $n_{\max}$ increases by $\alpha$ when $x$ increases by 1. Our full model is then:

$$\dot{x} = -\lambda x + \delta n, \tag{4a}$$

$$\dot{n} = rn\left(1 - \frac{n}{\alpha x + \beta}\right). \tag{4b}$$

## 3 Model analysis

We could not solve the model analytically,[1] but we have characterized its behavior as follows. From Eq. 4 we see that the conditions for demographic growth ($\dot{n} > 0$) and cultural growth ($\dot{x} > 0$) are, respectively:

$$n < \alpha x + \beta, \tag{5a}$$

$$n > \frac{\lambda}{\delta} x. \tag{5b}$$

These linear inequalities define regions in the $(x, n)$ plane where $x$ and $n$ increase, are stable (on the lines themselves), or decrease. That is, $n$ grows when the system finds itself *below* the line $n = \alpha x + \beta$, while $x$ grows *above* the line $n = \lambda x/\delta$. Given that we are only interested in positive $x$ and $n$, the two lines can partition the state space in two ways: either they intersect in the first quadrant (Fig. 2, left, corresponding to $\lambda > \alpha\delta$) or they do not (Fig. 2, right, $\lambda < \alpha\delta$). We show next that these geometrical arrangements correspond to two dynamical regimes of the model.

### 3.1 Demo-cultural equilibrium

We look for an equilibrium by setting $\dot{x} = 0$ and $\dot{n} = 0$ in (4) and solving for $x$ and $n$. This yields:

$$x^\star = \frac{\beta\delta}{\lambda - \alpha\delta}, \qquad n^\star = \frac{\beta\lambda}{\lambda - \alpha\delta}. \tag{6}$$

The point $(x^\star, n^\star)$ is a meaningful equilibrium if both $x^\star$ and $n^\star$ are positive, i.e., if $\lambda > \alpha\delta$ as anticipated above. We prove in the appendix that this equilibrium is stable. Considering the signs of $\dot{x}$ and $\dot{n}$, we also see that the equilibrium is a global attractor of system dynamics for any initial condition $(x_0, n_0)$ for which $n_0 \neq 0$ (Fig. 2, left; Fig. 3, left). Lastly, $(x^\star, n^\star)$ is independent of the demographic growth rate $r$; the latter only affects the rate and the trajectory by which equilibrium is approached.

### 3.2 Demo-cultural explosion

If $\lambda < \alpha\delta$, the two lines $n = \alpha x + \beta$ and $n = \lambda x/\delta$ do not intersect for positive $x$ and $n$, and partition the system state space into three regions (Fig. 2, right). Considering the signs of $\dot{x}$ and $\dot{n}$ in these regions one sees that, from any initial condition such that $n_0 > 0$, the system will eventually enter the region between the two lines, after which it will never exit from it (Fig. 2, right). In this region, we always have $\dot{x} > 0$ and $\dot{n} > 0$, so both culture and population size grow without bound (although, depending on initial conditions, either population size or amount of culture may temporarily decrease; Fig. 3, right). We show in the appendix that, in the long run, demo-cultural growth is exponential in time:

[1] In the case $\beta = 0$, which we are not considering in the main text, a solution is obtained noting that the variable $m = n/x$ obeys a logistic equation, as one can verify by using (4) to write $\mathrm{d}(n/x)/\mathrm{d}t = (\dot{n}x - n\dot{x})/x^2$ in terms of $m$.

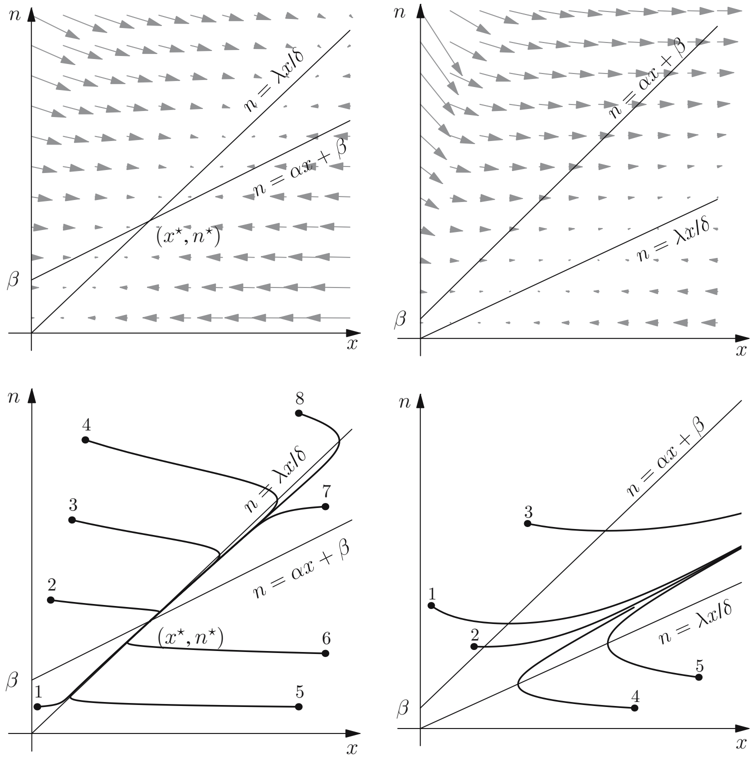


**Fig. 2** The stable regime ($\lambda > \alpha\delta$, *left column*) and explosive regime ($\lambda < \alpha\delta$, *right column*) of the model in (4), represented as velocity fields (*top row*) or system trajectories (*bottom row*) in the $(x, n)$ plane. The characteristic lines, Eq. 5, are also shown. *Top*: each arrow gives the direction of motion of the system at a given point; arrow length is proportional to speed of motion. *Bottom*: each line represents a system trajectory from the starting point marked with a bullet. Trajectories are labeled by numbers for comparison with Fig. 3

$$x(t) \simeq e^{Kt}, \tag{7a}$$

$$n(t) \simeq e^{Kt} \tag{7b}$$

with

$$K = r\frac{\alpha\delta - \lambda}{\alpha\delta + r}. \tag{8}$$

Long-term exponential increase of $x$ and $n$ at a common rate is shown in Fig. 3, right.

### 3.3 From extensive to intensive growth

According to Eq. 7, in the explosive regime of our model $n$ and $x$ grow at the same rate. That is, culture grows to the extent that the population grows, but the amount of culture per individual does not grow. This pattern is referred to as *extensive growth* by economic historians, while an increase in *x per individual*, i.e., an increase in the ratio $x/n$, would be described as *intensive growth*. Economic historians usually consider variables such as income or productivity rather than amount of culture, but the argument is the same (Jones 1996b). For instance, combining the data in the two panels of

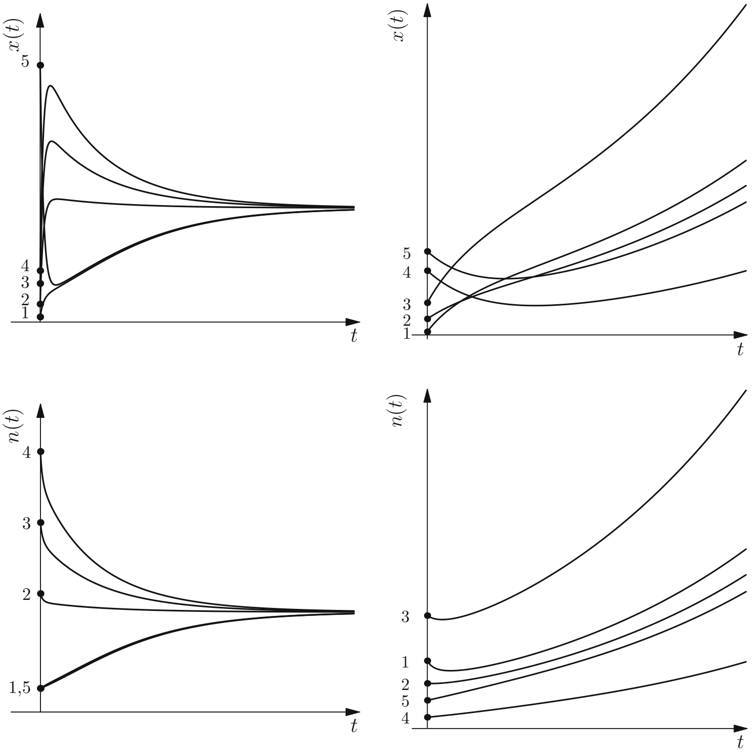


**Fig. 3** Amount of culture *x* (*top row*) and population size *n* (*bottom row*) as a function of time, in the equilibrium (*left column*) and explosive (*right column*) regimes of the model in Eq. 4. Note that, in the explosive regime (*right column*), both *x* and *n* increase exponentially, at the same rate, although initially they can either increase or decrease. The curves show some of the trajectories in Fig. 2, *bottom*, as shown by the numeric labels

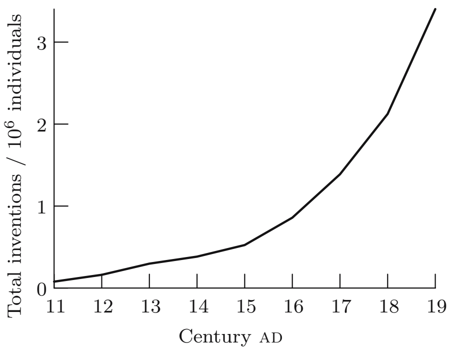


**Fig. 4** Exponential increase in the number of scientific and technological inventions *per individual*, obtained combining the data in the *left* and *right* panels of Fig. 1

Fig. 1 one can show that the ratio between scientific innovations and population size has been growing exponentialyy between 1100 and 1900 AD (Fig. 4).

In our model, intensive growth can be obtained modifying the assumption that the cultural output of a population is proportional to its size ($\delta n$ term in Eq. 4a), allowing cultural output to increase more rapidly with population size. A simple example is a quadratic increase:

$$\dot{x} = -\lambda x + \delta n^2. \tag{9}$$

We show in the appendix that this modified model has two regimes. In contrast with the previous model, in both regimes explosive growth is possible, and in both cases $x$ grows, in the long run, more quickly than $n$—the model exhibits intensive growth of culture. The difference between the two regimes is that one of them features a stable equilibrium point as well as the possibility for explosive growth, the fate of the system being determined by the starting point. The first regime applies if $\lambda < 4\alpha\delta\beta$, the second if $\lambda > 4\alpha\delta\beta$ (see Appendix). This condition is similar to the one we obtained before, $\lambda \gtrless \alpha\delta$, but also includes the environment's carrying capacity in the absence of culture, $\beta$. This means that a richer environment favors the transition to explosive growth. Lastly, we note that, in the second regime, a population can remain in equilibrium even if growth is, in principle, available to it; a fluctuation in population size or amount of culture can trigger growth by bringing the population outside the basin of attraction of the equilibrium point.

The modification of Eq. 4a into Eq. 9 has an interesting interpretation. In the former equation, the assumption that the cultural output of a population be proportional to population size means essentially that individuals create culture independently of each other. A stronger dependence of cultural output on population size can be derived instead from the assumption that individuals cooperate with each other in producing culture, so that two individuals together can produce more culture than the same two individuals independently of each other. This is consistent with the claim that improvements in social organization have been responsible for generating intensive growth in human history (Jones 1996a).

## 4 Discussion

We have shown that a transition from an equilibrium to an expanding population is possible in the presence of cumulative culture, even in a constant ecological environment. The condition for demo-cultural explosion, $\lambda < \alpha\delta$, has an appealing interpretation as $\lambda$ is the accuracy of cultural transmission, $\alpha$ is the effect of culture on ecological carrying capacity and $\delta$ is the amount of culture produced per unit time per individual. Thus, $\alpha\delta$ measures the positive feedback between culture and demography: demo-cultural explosion occurs when this feedback is strong enough to over-compensate the loss of culture due to faulty cultural transmission. The explosive regime of our model features exponential growth of both culture and population size, Eq. 7, which agrees with empirical data on long term demographic trends (Jones 1996b; McEvedy and Jones 1978) and cultural accumulation (Lehman 1947; Enquist and Ghirlanda 2006, Submitted). We have also developed the model to investigate the possibility of intensive growth of culture, showing that a transition from extensive to intensive growth can be brought about by a qualitative change in how individuals interact to produce culture, as hypothesized for economic growth by Jones (1996a).

We have here assumed for simplicity that all aspects of culture are beneficial to demographic growth. This is clearly unrealistic because culture can decrease the carrying capacity of the environment (e.g., by over-exploitation of resources) and also bring about traits that restrain fertility (e.g., urging individuals to postpone reproduction to achieve higher education). Modeling such complexity seems essential to investigate the ultimate fate of a population that exploits its environment, and also to understand a second kind of demographic transition: the reduction in fertility observed in many countries as living standards improve (Borgerhoff Mulder 1998). By allowing culture to both increase and decrease population growth, it may be possible to model both kinds of demographic transitions.

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## Appendix A: Stability of equilibria

The stability of equilibria can be assessed by calculating the eigenvalues of the so-called Jacobian matrix at the equilibrium point. For a system

$$\dot{x} = f(x, n), \tag{10a}$$

$$\dot{n} = g(x, n). \tag{10b}$$

The Jacobian is the matrix of partial derivatives

$$J(x, n) = \begin{pmatrix} \dfrac{\partial f}{\partial x} & \dfrac{\partial f}{\partial n} \\ \dfrac{\partial g}{\partial x} & \dfrac{\partial g}{\partial n} \end{pmatrix}. \tag{11}$$

In the present case we have

$$J(x, n) = \begin{pmatrix} -\lambda & \delta \\ \dfrac{\alpha r n^2}{(\alpha x + \beta)^2} & r - \dfrac{2rn}{\alpha x + \beta} \end{pmatrix}. \tag{12}$$

Which, evaluated at the equilibrium given in (6), is

$$J(x^\star, n^\star) = \begin{pmatrix} -\lambda & \delta \\ \alpha r & -r \end{pmatrix}. \tag{13}$$

The eigenvalues are

$$q_\pm = -\frac{\lambda + r}{2} \pm \frac{1}{2}\sqrt{(\lambda - r)^2 + 4r\alpha\delta}. \tag{14}$$

An equilibrium is asymptotically stable (stable and attracting nearby points) if (the real parts of) both eigenvalues are negative. Now, $q_-$ is surely negative (the square root argument is positive), while $q_+$ is negative if

$$\sqrt{(\lambda - r)^2 + 4r\alpha\delta} < \lambda + r. \tag{15}$$

Squaring and simplifying the resulting expression shows that this is equivalent to $\lambda > \alpha\delta$, the condition derived in the main text for the existence of an equilibrium. Thus, if the equilibrium $(x^\star,\ n^\star)$ exists, it is asymptotically stable.

When $\lambda < \alpha\delta$ the point $(x^\star,\ n^\star)$ is still, formally, an equilibrium, although it has no meaning in our model because $x$ and $n$ are negative. It is now and unstable equilibrium because $\lambda < \alpha\delta$ implies $q_+ > 0$. This change in stability underlies the transition between demo-cultural equilibrium and demo-cultural explosion discussed in the main text.

## Appendix B: rate of demo-cultural explosion

Because the system trajectory must lie between two lines (Fig. 2), we can conclude that long term system behavior in the explosive regime is characterized by a constant ratio between $n$ and $x$. Writing $n/x = k$ we can approximate the system equations by

$$\dot{x} \simeq (\delta k - \lambda)x, \tag{16a}$$

$$\dot{n} \simeq r\left(1 - \frac{k}{\alpha}\right)n. \tag{16b}$$

This approximation is valid for long-term behavior. Since this entails large values of $x$ and $n$, in the second equation we have neglected $\beta$. The value of $k$ can be obtained noting that $n/x = k$ implies equality of the coefficients in Eqs. 16a and 16b:

$$\delta k - \lambda = r\left(1 - \frac{k}{\alpha}\right) \tag{17}$$

or

$$k = \alpha\frac{r + \lambda}{r + \alpha\delta}. \tag{18}$$

Substituting this value in (16) we conclude that $\dot{x} \simeq Kx$ and $\dot{n} \simeq Kn$, with

$$K = r\frac{\alpha\delta - \lambda}{\alpha\delta + r}. \tag{19}$$

That is, both population size and amount of culture grow exponentially in time at the rate in equation (19).

## Appendix C: intensive growth

The analysis of the model consisting of Eqs. 4b and 9 follows the same lines as the first one. The condition for cultural growth is now

$$n > \sqrt{\frac{\lambda}{\delta}x}, \tag{20}$$

which means that the line through the origin in Fig. 2 is replaced by a square-root curve. The condition for demographic growth is unchanged, and corresponds to the same line as before. The line and the curve can intersect in 0 or 2 points. Transition between these two regimes corresponds to the line lying tangent to the curve. The system of equations $\dot{x} = 0, \dot{n} = 0$ has now two pairs of solutions:

$$x_\pm = \frac{1}{2\alpha}(1 \pm \sqrt{R}), \tag{21a}$$

$$n_{\pm} = \frac{\lambda}{2\alpha\delta}(1 \pm \sqrt{R}) \tag{21b}$$

with

$$R = 1 - \frac{4\alpha\beta\delta}{\lambda}. \tag{22}$$

If $R > 0$ (or $\lambda > 4\alpha\beta\delta$) all of $x_{\pm}$ and $n_{\pm}$ are positive and thus both $(x_-, n_-)$ and $(x_+, n_+)$ are meaningful equilibria. Analysis of the approximating linear system, with the same technique as above, shows that the equilibrium closer to the origin is always stable, the other always unstable. Thus, $x$ and $n$ either go toward an equilibrium value or grow without bound. If $R < 0$ there are no equilibria for positive $x$ and $n$, and a simple qualitative analysis similar to the one of Fig. 2, right, establishes that $x$ and $n$ will grow without bound.

An easy way to see that the model exhibits intensive growth is to calculate the derivative of $n/x$ using Eq. 9 and 4b:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{n}{x} = \frac{\dot{n}x - n\dot{x}}{x^2} = \left(\frac{2}{\alpha} - \delta n\right)\left(\frac{n}{x}\right)^2 + (2+\lambda)\frac{n}{x}. \tag{23}$$

For large enough $n$ this quantity turns negative, meaning that in the long run $x$ grown faster than $n$.

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